Solving Equations Graphically – Is There a Solution?

Objective:
Students will solve equations algebraically and graphically then connect the solutions to the graphs.

Connections to Previous Learning:
Students should be familiar with graphing functions with and without technology, solving equations, and parent graphs.

Connections to AP*:
AP Calculus Topic: Analysis of Functions

Materials:
Student Activity pages, graphing calculators

When solving the equation, $-2x+2=3x-3$, the students will graph the following functions then find the point of intersection.

$$y=-2x+2$$
$$y=3x-3$$

The x coordinate is the value for x that makes the left side of the problem equal the right side. The y coordinate is the value of each side of the equation.

It is important for students to understand that the algebraic techniques used to solve equations will change the picture of the problem. When solving $\sqrt{x-1}=-2x+5$, the problem asks for the point(s) of intersection of a root curve and a line. To solve for x, students are taught to square both sides. This changes the problem into the intersection of a line and a parabola. While the solutions for x may be the same, they may also pick up extra solutions. Students are empowered when they know what the picture of the problem looks like.

Solving Equations Graphically – Is There a Solution?

I. Solve each equation by graphing, without technology then solve each equation algebraically.

1. $-2x+2=3x-3$
2. $2x-2=-\frac{2}{3}x+6$
3. $x+5=-\frac{3}{2}x$
4. $|x-2|=3$
5. $-|x-1|+2=3$
6. $-x^2+2=-2$
7. $x^2+1=-\frac{1}{2}x-2$

II. Solve each equation graphically, without technology then check your answer by graphing on a calculator.

8. $|x+2|=\frac{1}{3}x+2$
9. $(x-2)^2+1=-x+5$
10. $x^2=\sqrt{x}$
11. $-(x+1)^3=-x-1$

III. Use the equation $\sqrt{x-1}=-2x+5$ to answer the following questions.

a) Solve the given equation algebraically. Did your method produce any extra solutions?

b) Solve the given equation graphically.

c) Graph $y=-\sqrt{x-1}$ on the same grid you used in part b. Use this graph and the graph of $y=-2x+5$ to help you locate the extra solution on the grid. Explain the meaning of the extra solution in the context of the graph.